The energy dependence for the components of the energy release from fission are provided in ENDF-102, Section 1.5 [Herman]. The values for $\delta E_i(E_{\text{inc}})$ are

$$\begin{aligned}
&\delta ET = -1.057 E_{\text{inc}} + 8.07\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right],\\
&\delta EB = \delta EGD = 0.075 E_{\text{inc}},\\
&\delta ENU = 0.100 E_{\text{inc}},\\
&\delta ENP = -1.307 E_{\text{inc}} + 8.07\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right],\\
&\delta EFR = \delta EGP = 0.
\end{aligned} \qquad \text{Eq. (1)}$$

However, on inspection of these definitions it is apparent that the units of the coefficient 8.07 have been inadvertently omitted and appear to be in MeV. This makes sense since this value represents the excess mass of a neutron, i.e., ~8.07 MeV. Therefore, the term should be $8.07\text{x}10^6$ eV, since default units in ENDF-102 are eV.

The ENDF/B-VII.0 *Nuclear Data Sheets* article [Chadwick] provides the same energy definitions but with the following changes:

$$\begin{aligned}
&\delta ER = -1.057 E_{\text{inc}} - 8.07\times 10^6\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right],\\
&\delta EB = \delta EGD = 0.075 E_{\text{inc}},\\
&\delta ENU = 0.100 E_{\text{inc}},\\
&\delta ENP = -1.307 E_{\text{inc}} - 8.07\times 10^6\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right],\\
&\delta EFR = \delta EGP = 0.
\end{aligned} \qquad \text{Eq. (2)}$$

These definitions have corrected the obvious error in the magnitude of the second term in $\delta ENP$ and $\delta ER$, but have introduced a sign change on the same term and switched $\delta ET$ with $\delta ER$. Even if $\delta ER$ is accepted in lieu of $\delta ET$, the formula should be changed to -1.157 $E_{\text{inc}}$, which would be incorrect, as well.

Some references [MacFarlane, Robinson] provide a definition for the energy dependence of the fission fragment energy release where $\delta EFR = 0.057\ E_{\text{inc}}$. However, this term is set to zero in the definitions shown in Herman and Chadwick.

Both Sher and Robinson cite Walker as the source of the energy-dependence of the components of fission energy release equations. Walker defines the total, delayed and prompt components of the energy release from fission, induced by a neutron of energy $E_{\text{inc}}$, as:

$$\begin{aligned}
&ET\left(E_{\text{inc}}\right) = Q_{\text{G}}\left(E_{\text{inc}}\right) + E_{\text{inc}},\\
&ED\left(E_{\text{inc}}\right) = EB\left(E_{\text{inc}}\right) + EGD\left(E_{\text{inc}}\right) + ENU\left(E_{\text{inc}}\right) + END\left(E_{\text{inc}}\right),\\
&EP\left(E_{\text{inc}}\right) = EFR\left(E_{\text{inc}}\right) + EGP\left(E_{\text{inc}}\right) + ENP\left(E_{\text{inc}}\right),
\end{aligned} \qquad \text{Eq. (3)}$$

where nomenclature is consistent with that already defined, $Q_{\text{G}}$ is the energy released from fission, and the quantities $ED(E_{\text{inc}})$ and $EP(E_{\text{inc}})$ represent the delayed- and prompt-energy release from fission, respectively.

Walker goes on to assert that the energy released from fission, $Q_{\text{G}}$, is the most fundamental of all these quantities and the only one that can be calculated accurately using mass balance:

$$Q_{\text{G}}\left(E_{\text{inc}}\right) = XF - XFP\left(E_{\text{inc}}\right) - \left[\bar{\nu}\left(E_{\text{inc}}\right) - 1\right] XN, \qquad \text{Eq. (4)}$$

where $XF$ is the excess mass of the fissionable material, $XFP$ is the average fission product mass excess, and $XN$ is the excess mass of a neutron, 8.07 MeV.

Walker states that because fission product distributions are centered on the minimum of the mass excess vs. mass curve, and because the minimum is very broad, $XFP(E_{\mathrm{inc}})$ turns out to be virtually independent of the fissioning nuclide and only slightly dependent on $E_{\mathrm{inc}}$. Thus the values obtained using Eq. (4) can be closely approximated by

$$Q_{\mathrm{G}}\left(E_{\mathrm{inc}}\right) = 210.1 + 2.05\left(AF - 233\right) - 8.07\left[\bar{\nu}\left(E_{\mathrm{inc}}\right) - 1\right] + 0.057E_{\mathrm{inc}}, \qquad \text{Eq. (5)}$$

where terms have units of MeV and *AF* is the mass number of the fissioning material.

Walker defines the difference between $ET(E_{\mathrm{inc}})$ and $ET(0)$ be

$$\delta ET\left(E_{\mathrm{inc}}\right) = ET\left(E_{\mathrm{inc}}\right) - ET\left(0\right) = Q_{\mathrm{G}}\left(E_{\mathrm{inc}}\right) + E_{\mathrm{inc}} - Q_{\mathrm{G}}\left(0\right). \qquad \text{Eq. (6)}$$

Substituting from Eq. (6) gives

$$\delta ET\left(E_{\mathrm{inc}}\right) = 1.057E_{\mathrm{inc}} - 8.07\left[\bar{\nu}\left(E_{\mathrm{inc}}\right) - \bar{\nu}\left(0\right)\right]. \qquad \text{Eq. (7)}$$

Walker provides a table in Appendix 2 of the 1979 CSEWG minutes (presumably from AECL-5259) showing data for several fissionable nuclides. Using these data, several observations can be made: 1) the total energy release from fission using Eq. (5) agrees very well to that calculated by mass balance, 2) the delayed component of the energy release from fission decreases with increasing incident neutron energy, and 3) the prompt component of the energy release from fission increases with increasing incident neutron energy.

Further, if the components of the delayed-energy release data are plotted and fit using a linear relationship, the slopes are nearly the same and are consistent with the magnitudes of the values for $\delta EB$, $\delta EGD$, and $\delta ENU$, suggested in ENDF-102.

The only significant contributor to the energy dependence of the total prompt energy release from fission is due to the prompt neutron term. The change in the prompt neutron energy release as a function of energy is proportional to the total prompt energy release. This supports setting the $\delta EGP$ and $\delta EFR$ terms to zero, as suggested in ENDF-102.

Therefore, the following relationships can be defined:

$$\begin{aligned}
\delta EB\left(E_{\mathrm{inc}}\right) &= EB\left(E_{\mathrm{inc}}\right) - EB\left(0\right) = -0.075E_{\mathrm{inc}},\\
\delta EGD\left(E_{\mathrm{inc}}\right) &= EGD\left(E_{\mathrm{inc}}\right) - EGD\left(0\right) = -0.075E_{\mathrm{inc}},\\
\delta ENU\left(E_{\mathrm{inc}}\right) &= ENU\left(E_{\mathrm{inc}}\right) - ENU\left(0\right) = -0.100E_{\mathrm{inc}},
\end{aligned} \qquad \text{Eq. (8)}$$

and

$$\delta ENP\left(E_{\mathrm{inc}}\right) = \begin{cases} \delta ET\left(E_{\mathrm{inc}}\right) - \delta ED\left(E_{\mathrm{inc}}\right), \\ 1.057E_{\mathrm{inc}} - 8.07\left[\bar{\nu}\left(E_{\mathrm{inc}}\right) - \bar{\nu}\left(0\right)\right] + 0.250E_{\mathrm{inc}}, \\ 1.307E_{\mathrm{inc}} - 8.07\left[\bar{\nu}\left(E_{\mathrm{inc}}\right) - \bar{\nu}\left(0\right)\right]. \end{cases} \qquad \text{Eq. (9)}$$

These definitions match exactly the definitions provided in ENDF-102, recognizing that in Walker's definitions

$$\delta E_i\left(E_{\mathrm{inc}}\right) = E_i\left(E_{\mathrm{inc}}\right) - E_i\left(0\right), \qquad \text{Eq. (10)}$$

and ENDF-102 reverses the sign:

$$\delta E_i\left(E_{\text{inc}}\right) = E_i\left(0\right) - E_i\left(E_{\text{inc}}\right). \qquad \text{Eq. (11)}$$

Values given by in ENDF-102 are in complete agreement with the values determined by Walker, and all that is required is to change the units from MeV to eV. Therefore, based on this analysis using Walker's derivations, the final set of recommended equations for ENDF-102 is

$$\begin{aligned}
&\delta EB = \delta EGD = 0.075 E_{\text{inc}}, \\
&\delta ENU = 0.100 E_{\text{inc}}, \\
&\delta EFR = \delta EGP = 0.0, \\
&\delta ENP = -1.307 E_{\text{inc}} + 8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right], \\
&\delta ET = -1.057 E_{\text{inc}} + 8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right].
\end{aligned} \qquad \text{Eq. (12)}$$

However, the definitions provided in Chadwick are inconsistent with that provided in Walker. Specifically, the following changes to Eq. (2) are required to be consistent with Eq. (12):

$$\begin{aligned}
&\delta E\cancel{R}T = -1.057 E_{\text{inc}} \cancel{-} +8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right], \\
&\delta ENP = -1.307 E_{\text{inc}} \cancel{-} +8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right].
\end{aligned} \qquad \text{Eq. (13)}$$

For computing the energy release by the energy-balance method in NJOY/HEATR [MacFarlane] an "effective-prompt-energy-release-from-fission-$Q$ value" is computed using the fission-$Q$ value, $Q_{\text{ENDF}}$, provided in ENDF (MF=3, MT=18). $Q_{\text{ENDF}}$ is referred to as a pseudo-$Q$ value in that it is defined to be the total energy release less the neutrino energy. To compute the effective-prompt-energy-release-from-fission-$Q$ value, $Q'$, NJOY/HEATR first subtracts the delayed-components of energy release, *EB* and *EGD* (energy released as delayed neutrons is again assumed to be negligible), and the incident neutron energy from $Q_{\text{ENDF}}$ giving

$$Q'\left(E_{\text{inc}}\right) = \begin{cases} ET\left(0\right) - \delta ET - \left(ENU\left(0\right) - \delta ENU\right) - \left(EB\left(0\right) - \delta EB\right) \\ -\left(EGD\left(0\right) - \delta EGD\right) - E_{\text{inc}}, \end{cases}$$

$$Q'\left(E_{\text{inc}}\right) = \begin{cases} ET\left(0\right) + 1.057 E_{\text{inc}} - 8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right] - ENU\left(0\right) + 0.100 E_{\text{inc}} \\ -EB\left(0\right) + 0.075 E_{\text{inc}} - EGD\left(0\right) + 0.075 E_{\text{inc}} - E_{\text{inc}}, \end{cases}$$

$$Q'\left(E_{\text{inc}}\right) = ET\left(0\right) - ENU\left(0\right) - EB\left(0\right) - EGD\left(0\right) + 0.307 E_{\text{inc}} - 8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right], \qquad \text{Eq. (14)}$$

but since $Q_{\text{ENDF}}$ is defined to be $ET(0) - ENU(0)$,

$$Q'\left(E_{\text{inc}}\right) = Q_{\text{ENDF}} - EB\left(0\right) - EGD\left(0\right) - 8.07\times 10^{6}\left[\bar{\nu}\left(E_{\text{inc}}\right) - \bar{\nu}\left(0\right)\right] + 0.307 E_{\text{inc}}. \qquad \text{Eq. (15)}$$

An equivalent way to calculate this is to sum just the prompt components of energy release, as is shown in Chadwick:

$$\begin{aligned}
&Q'\left(E_{\text{inc}}\right) = EFR\left(0\right) - \delta EFR + ENP\left(0\right) - \delta ENP + EGP\left(0\right) - \delta EGP - E_{\text{inc}}, \\
&Q'\left(E_{\text{inc}}\right) = EFR\left(0\right) + ENP\left(0\right) + EGP\left(0\right) - 8.07\times 10^{6}\left[\bar{\nu}\left(E\right) - \bar{\nu}\left(0\right)\right] + 0.307 E_{\text{inc}}.
\end{aligned} \qquad \text{Eq. (16)}$$

By inspection of the code in NJOY99.259, the HEATR module computes $Q'(E)$ using

$$Q'_{\mathrm{NJOY}}\left(E_{\mathrm{inc}}\right)=Q_{\mathrm{ENDF}}-EB\left(0\right)-EGD\left(0\right)-8.07\times10^{6}\left[\bar{\nu}\left(E_{\mathrm{inc}}\right)-\bar{\nu}\left(0\right)\right]+0.307E_{\mathrm{inc}}, \quad \text{Eq. (17)}$$

and is only correct if $\delta EFR = 0.0$, as specified in Eq. (2) and Eq. (3). This is particularly puzzling given that the NJOY manual states that $\delta EFR$ is taken to be $0.057E_{\mathrm{inc}}$.

**REFERENCES**


[**Chadwick**] M.B. Chadwick, *et al*, "ENDF/B-VII.0: Next Generation Evaluated Nuclear Data Library for Nuclear Science and Technology," *Nuclear Data Sheets*, **107**, pp. 2931-3060 (2006).

[**Herman**] M. Herman and A. Trkov, "ENDF-6 Formats Manual, Data Formats and Procedures for the Evaluated Nuclear Data File ENDF/B-VI and ENDF/B-VII," Brookhaven National Laboratory, Upton, NY, Report BNL-90365-2009, CSEWG Document ENDF-102 (2009).

[**MacFarlane**] R. E. MacFarlane, D. W. Muir, "The NJOY Nuclear Data Processing System," Los Alamos National Laboratory, Los Alamos, NM, LA-12740-M (1994).

[**Robinson**] G. Robinson, "NJOY Treatment of Energy Release in Fission," NJOY/ROBINSON-01 – 11/19/1993.

[**Sher**] R. Sher, C. Beck, "Fission Energy Release for 16 Fissioning Nuclides," Stanford University, EPRI Report NP-1771 (1981).

[**Walker**] W. H. Walker, "Energy Per Fission for ENDF/B-V," Minutes of CSEWG Meeting, Enclosure 5, Appendix I, Brookhaven National Laboratory, NY, October 31 – November 1, 1979.